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The analysis of the frictional effect on stress - strain data from uniaxial compression of cheese

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Uniaxial compression tests were performed on Gruyere and Mozzarella cheeses. It was observed that shorter samples appeared stiffer when no lubrication was used. This dependence on sample height was eliminated when a synthetic grease lubricant with polytetrafluorethylene (PTFE) was used. Therefore, the true stress-strain curves, i.e. free of frictional effects, were determined. Methods for reproducing these curves using data from unlubricated tests were then sought. It was shown that the true stress-strain curves can be determined by testing samples of increasing heights until the difference between consecutive curves is negligible. The curve corresponding to the tallest sample can then be taken to represent the true stress-strain curve. If size or shape limitations do not allow testing of sufficiently tall samples, quadratic extrapolation of the results may be performed. Alternatively, an iterative finite element analysis could be used. The latter is a more accurate but more time consuming method than the extrapolation procedure. In addition it requires that the coefficient of friction, μ, is known. It was shown that the latter can be derived from an analytical scheme. These values of μ were approximately 0.1 for Gruyere and 0.3 for Mozzarella and they were in close agreement with numerical predictions.

1. Introduction

Uniaxial compression tests on cylindrical or cubic samples are widely used to determine the mechanical behaviour of various materials. The usual test procedure involves compressing the sample between two platens which approach each other at a constant speed. The compressive load and corresponding displacement are recorded and are post processed to give the stress - strain relationship of the material.

The drawback of this test is that friction between the sample and the loading platens can lead to an inhomogeneous stress - strain state in the sample. Evidence of this effect taking place is the barrel shape of the loaded specimen. If compression is performed under conditions where there is no friction, the deformation is homogeneous and the sample retains its cylindrical shape. When the end faces of the sample are restricted from spreading because of friction, the material adjacent to the loading platens resists deformation, as opposed to the central portion of the specimen. The effect of these partially deformed zones is more pronounced in shorter specimens because of the overall smaller specimen volume. This explains why, to achieve the same compression in two specimens of different heights but of equal cross sectional area, a larger stress is required for the shorter sample.

On the other hand, the advantage of the compression test over the tensile test is that it avoids the need for cutting complex dumbbell shapes and for gripping the sample. This is especially useful in studies of mechanical properties of very soft materials such as foods. Indeed, for food materials, uniaxial compression tests have been widely used in studies whose aim is to establish correlations between the mechanical behaviour and texture, e.g. [1, 2]. This is because, at present, sensory evaluations are the usual means of texture characterisation. This is a highly variable method, partly due to the inherent subjectiveness of the method and partly due to the loosely defined and abstract terms used in the evaluation process. Mechanical properties provide an alternative method of characterisation which benefits from precise engineering terms and analysis. Specifically for cheese, the International Dairy Federation has produced a report which details many aspects of compression testing and the relation between instrumental and sensory evaluation [3].

The aim of this study was to investigate the frictional effect on the stress - strain data derived from uniaxial compression tests on cheese. More specifically, methods for quantifying the frictional conditions as well as determining the true stress - strain curve from experimental data influenced by friction were sought. Both analytical and numerical solutions to the problem were considered and verified using experimental data.

2. Experiments

The materials used were Processed Mozzarella (23% fat, 16.75% protein, 50% moisture) and Gruyere (32% fat, 28% protein, 34% moisture). These cheeses were chosen as they are contrasting in mechanical properties i.e. Gruyere is stiffer and fractures at smaller strains than Mozzarella. The Mozzarella cheese was supplied by The Pillsbury Company, USA, in blocks weighing approximately 0.4 kg while a 1.8 kg block of Gruyere was bought from a local supermarket. A wire cutter was first used to cut slices from the blocks, taking care to ensure that the slice thickness was uniform. From these slices, cylindrical samples of 20 mm diameter were cut using a borer.

In order to study the effect of friction, various heights were tested; 5, 8, 11, 15 and 20 mm for Gruyere and 8, 11, 15 and 20 mm for Mozzarella. These are nominal dimensions. The exact height of each sample was recorded within ±0.5 mm prior to testing. For each height a minimum of three replicate samples were tested.

The experiments were performed at room temperature using a 4466 Instron testing machine with a 1 kN load cell. The crosshead speed which was constant during any one test, was adjusted according to the sample height such that all samples were tested at the arbitrarily chosen initial strain rate of 0.72 min^{-1}. Therefore the crosshead speeds were 3.6, 5.8, 7.9, 10.8 and 14.4 mm/min for the heights of 5, 8, 11, 15 and 20 mm respectively. Two frictional conditions were examined by performing two series of tests for each cheese. Firstly, tests were performed where no lubricant was applied to the loading platen - sample interface before testing. Secondly, Superlube (Loctite Corporation), a multi-purpose synthetic grease lubricant with polytetrafluorethylene (PTFE), was spread in a thin layer on the loading platens prior to testing.

Corresponding values of load, P, and deflection, δ, were recorded on a PC connected to the testing machine. These were used to calculate the mean stress, p, and strain, ε, from:

$$p = \frac{Ph}{\pi R^2 H} \tag{1}$$

and

$$\varepsilon = -\ln \frac{h}{H} \tag{2}$$

where H is the original height, R is the original radius and h is the current height ($= H - \delta$). Note that Equation 1 assumes a constant volume deformation. This is a reasonable assumption for cheese and makes computations simpler [4]. The strain as defined in Equation 2 is the true or Hencky strain and for large deformations is a better estimate of the real strain in the sample than engineering strain.

During the tests, cracks were visible for Gruyere around the time when the maximum point in the stress - strain curve was reached. For Mozzarella, the experiments were stopped before cracks were observed.

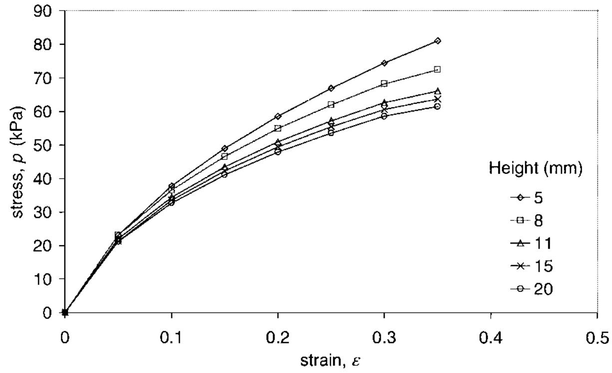


Figure 1 Average stress strain curves for non lubricated Gruyere samples.

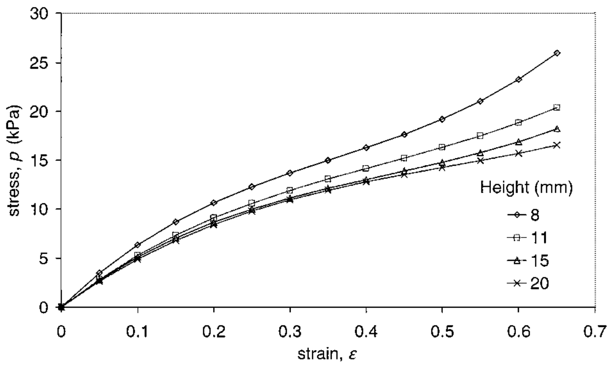


Figure 2 Average stress strain curves for non lubricated Mozzarella samples.

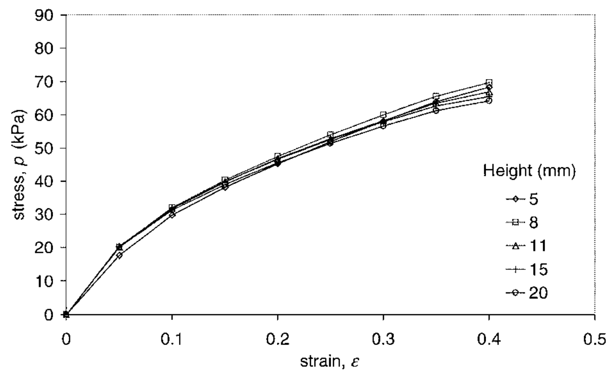


Figure 3 Average stress strain curves for lubricated Gruyere samples.

The stress - strain curves for Gruyere and Mozzarella when no lubrication was used are shown in Figs 1 and 2 respectively. For each nominal height, an average curve obtained from all samples is shown. The height effect is evident; shorter samples led to a higher stress - strain curve, i.e. the material appears stiffer. The results when Superlube was used are shown in Figs 3 and 4. The effectiveness of this lubricant in eliminating frictional effects for both cheeses is obvious as all heights now lead to a single stress - strain curve. Therefore the curves in Figs 3 and 4 can be assumed to represent the true stress - strain curves of the two cheeses at the specified test conditions.

Photographs of samples during testing with and without lubrication are shown in Figs 5 and 6 for Gruyere and Mozzarella respectively. All samples had an initial height of 8 mm. As expected from the data plotted in

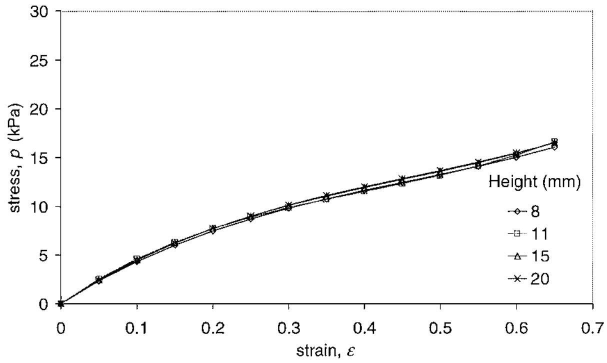
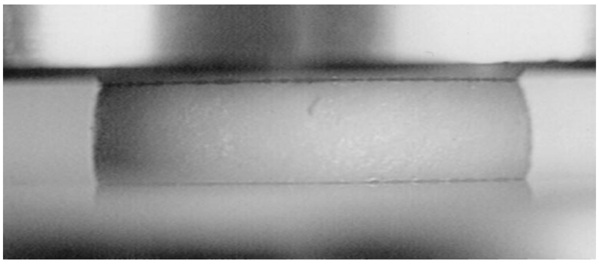
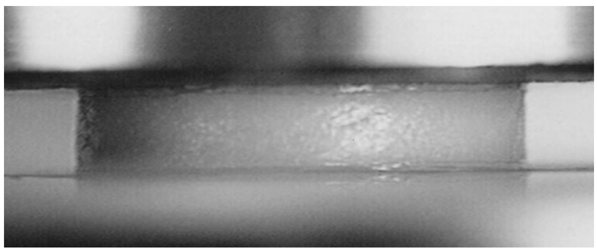


Figure 4 Average stress strain curves for lubricated Mozzarella samples.

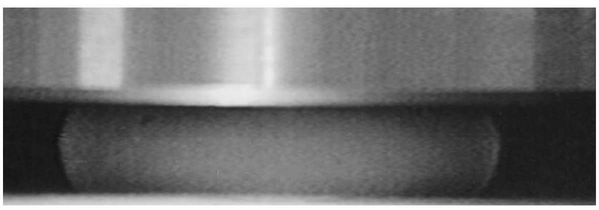


(a)

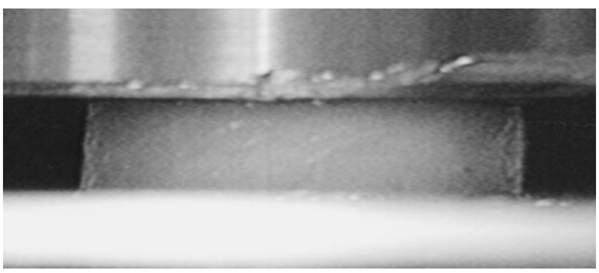


(b)

Figure 5 Deformed shape of Gruyere samples (a) unlubricated test (b) lubricated test.



(a)



(b)

Figure 6 Deformed shape of Mozzarella samples (a) unlubricated tests (b) lubricated tests.

Figs 1–4, the samples barrelled when no lubricant was used. However, the samples kept their cylindrical shape when Superlube was used which is evidence of uniform deformation taking place.

3. Analytical Solution

In this study, it was possible to eliminate frictional effects by using Superlube as a lubricant. However, there might be other materials for which friction can not be eliminated completely, e.g. bread dough. The data derived from the compression experiments would then be influenced by friction. A scheme which enables the calculation of the true stress-strain curve from such data would therefore be very useful. In this section a scheme based on analytical solutions will be investigated.

Friction between the workpiece and forming tools has been an important consideration in metalworking for many years. As a result, theoretical analyses of the compression of a flat circular disk can be found in many textbooks, e.g. [5, 6]. The analysis considers equilibrium of the forces acting on the disk, as shown in Fig. 7. The following simplifying assumptions are made: (i) there is no barrelling of the edges of the disk, and (ii) the thickness of the disk is small enough so that the axial compressive stress σ_z is constant through the thickness. Coulomb friction is assumed. The following expression for σ_z as a function of radial distance r is then derived [5]:

$$\sigma_z = \sigma_0 e^{\frac{2\mu}{h}(a-r)} \tag{3}$$

where μ is the coefficient of friction and σ_0, a and h are corresponding values of the yield stress, radius and height of the compressed disk.

This pressure distribution is symmetrical about the centreline and rises to a sharp peak at the centre of the disk. For this reason it is often called a friction hill. The average stress, p, acting on the disk is then obtained by integration:

$$p = \frac{\int_0^a 2\pi\sigma_z r\,\mathrm{d}r}{\pi a^2} = \frac{\sigma_0}{2}\left(\frac{h}{\mu a}\right)^2\left[e^{\frac{2\mu a}{h}} - \frac{2\mu a}{h} - 1\right] \tag{4}$$

Using the constant volume assumption i.e. $\pi a^2 h = \pi R^2 H$ and Equation 2, Equation 4 becomes:

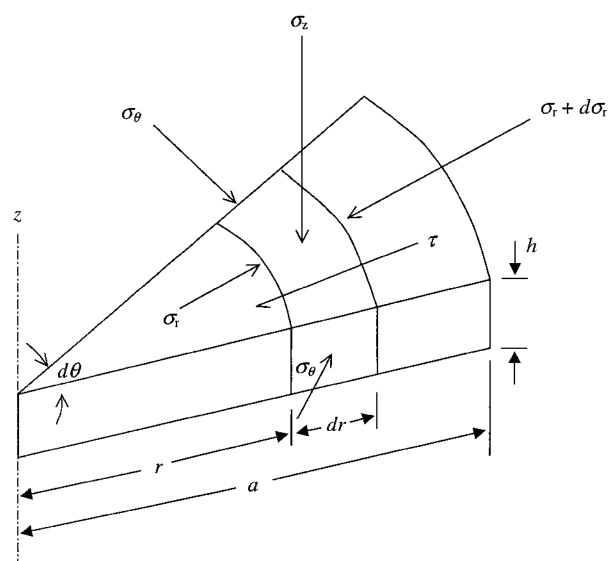


Figure 7 Stresses on a section of a compressed disk.

$$p = \frac{\sigma_0}{2}\left(\frac{H}{\mu R}e^{-\frac{3}{2}\varepsilon}\right)^2\left[e^{\frac{2\mu R}{H}e^{\frac{3}{2}\varepsilon}} - \frac{2\mu R}{H}e^{\frac{3}{2}\varepsilon} - 1\right] \quad (5)$$

which now gives p as a function of μ, the initial sample dimensions and applied strain.

The term $e^{2\mu a/h}$ in Equation 4 is usually approximated using the Maclaurin series expansion [7, 8]:

$$e^{\frac{2\mu a}{h}} = 1 + \frac{2\mu a}{h} + \frac{1}{2!}\left(\frac{2\mu a}{h}\right)^2 + \frac{1}{3!}\left(\frac{2\mu a}{h}\right)^3 + \cdots + \frac{1}{n!}\left(\frac{2\mu a}{h}\right)^n \quad (6)$$

When only the first four terms of the series are used and substituted in Equation 4 the result is:

$$p = \sigma_0\left(1 + \frac{2}{3}\frac{\mu a}{h}\right) = \sigma_0\left(1 + \frac{2}{3}\frac{\mu R}{H}e^{\frac{3}{2}\varepsilon}\right) \quad (7)$$

Therefore, from Equation 7, if p is plotted against $1/H$ for constant values of ε, the data should fall on straight lines, whose intercepts with the p axis will give the true yield stresses as a function of strain. Furthermore, the values of μ can be calculated from the slopes of the lines. This is very similar to the Cook and Larke procedure [9] where the mean stress p is extrapolated to an infinite height, i.e. $1/H = 0$. The argument behind this procedure is that friction effects would be negligible for an infinitely tall sample. The same procedure was used by the authors in earlier studies of mechanical properties of cheese [7, 10].

If one more term is retained in the series expansion, i.e. $n = 4$ in Equation 6, and the result is substituted in Equation 4, the following alternative relationship is obtained:

$$p = \sigma_0\left[1 + \frac{2}{3}\frac{\mu a}{h} + \frac{1}{3}\left(\frac{\mu a}{h}\right)^2\right] = \sigma_0\left[1 + \frac{2}{3}\left(\frac{\mu R}{H}e^{\frac{3}{2}\varepsilon}\right) + \frac{1}{3}\left(\frac{\mu R}{H}e^{\frac{3}{2}\varepsilon}\right)^2\right] \quad (8)$$

It is clear that for small values of μ and ε, the last term in Equation 8 can be ignored and Equation 7 is recovered. Equation 8 is a quadratic polynomial in $1/H$.

Equations 5, 7 and 8 were fitted to the data obtained from the experiments without lubrication. From the coefficients of the curve fit terms, values of σ_0 and μ as a function of ε were derived.

Fig. 8 shows plots of p versus $(1/H)$ for constant values of ε, for Mozzarella. Lines were fitted to the data and the accuracy of the approximation seems reasonable. A similar level of accuracy was also observed for Gruyere. The same data were also approximated using Equations 5 and 8. The extrapolated stress is plotted as a function of strain for all three cases in Figs 9 and 10. The experimental results from the lubricated tests are also plotted for comparison purposes. It is evident that all equations lead to stresses which are underestimated, with Equation 7 being the worst of the three. Equations 5 and 8 lead to very similar estimates which implies that higher order terms in the Maclaurin series in Equation 6 are negligible. The difference between the estimates from Equation 7 and those from Equation 8 increase with increasing strain. This is expected as the last term in Equation 8 increases with strain and therefore can not be ignored. The disagreement between the lubricated curves and the analytical extrapolations increases with strain for both cheeses. This could be due to an error introduced by the simplifying assumption that there is no barrelling since the latter is more pronounced for large strains.

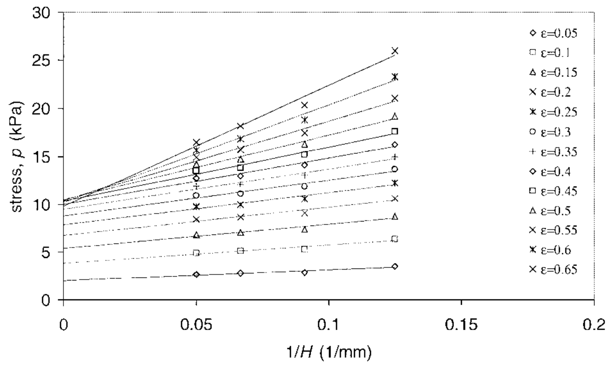


Figure 8 Stress versus $1/H$ for unlubricated Mozzarella samples at different strains.

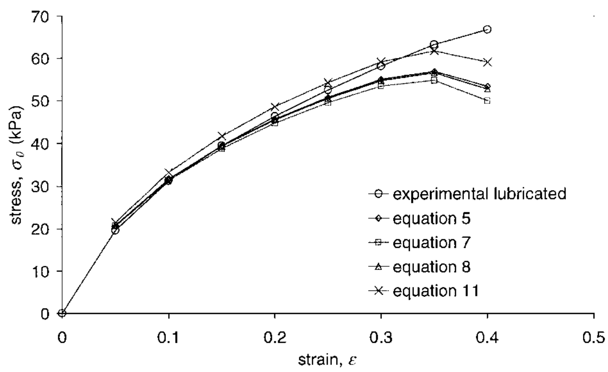


Figure 9 Comparison between extrapolated stress and experimental lubricated data for Gruyere.

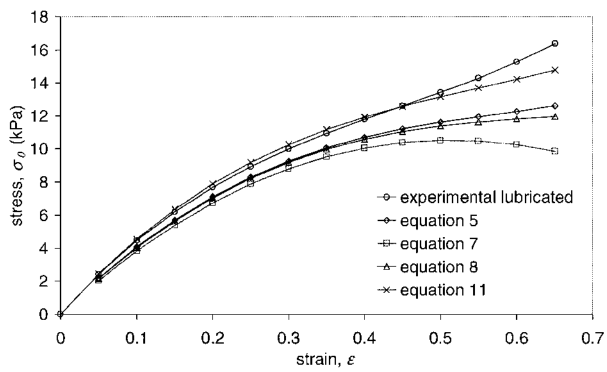


Figure 10 Comparison between extrapolated stress and experimental lubricated data for Mozzarella.

Estimates for the coefficient of friction for the three cases are shown in Figs 11 and 12. For both cheeses, Equation 7 leads to higher μ values than Equations 5 and 8. The latter lead to similar results for reasons explained above. A variation of μ with ε is observed for both cheeses. Nevertheless, for Gruyere, apart from the

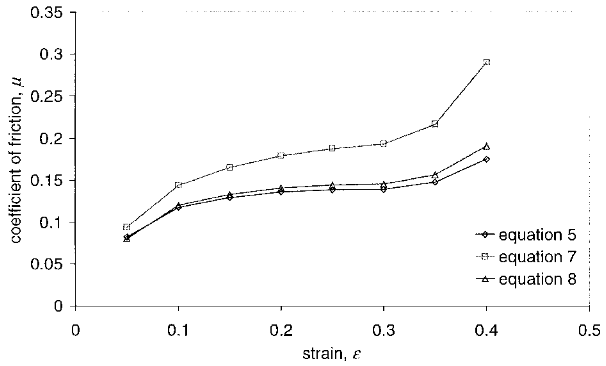


Figure 11 Predicted values of μ from analytical schemes for Gruyere.

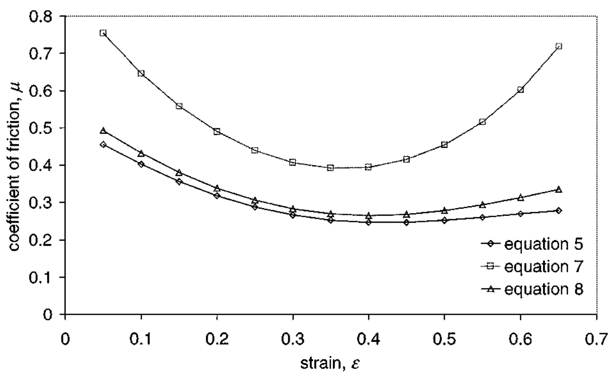


Figure 12 Predicted values of μ from analytical schemes for Mozzarella.

first and last point, μ is roughly constant at a value of 0.13 when either Equation 5 or 8 are used. For Mozzarella, μ levels off to approximately 0.3 for strains larger than 0.2.

4. Numerical modelling

The ABAQUS commercial finite element software package [11] was used to model the compression of a cylindrical sample between two flat, rigid platens. The model is axisymmetric and includes the top half of the cylinder only, since the middle surface is a plane of symmetry. Four noded quadrilateral elements were used.

The boundary conditions were symmetry on the cylinder axis ($r = 0$) and symmetry about the $z = 0$ line. A reference node on the rigid surface was displaced in the z direction such that the maximum imposed strains matched those measured in the experiments.

The material was assumed to behave as a linear elastic - plastic material. The onset of plasticity, usually defined by the point of non-linearity on the stress - strain curves, was taken to be at 5% strain for both cheeses. Work hardening was used to accommodate the rise in stress beyond the point of first yield. Incompressible behaviour i.e. a Poisson's ratio of 0.5 was assumed. A value of 0.49 was used in the finite element model instead to avoid possible numerical problems associated with truly incompressible behaviour [11].

4.1. Determination of stress - strain curves (Iterative finite - element procedure)

A procedure for determining the stress - strain curve from unlubricated test data via an iterative finite element analysis was proposed by Parteder and Bunten [12] in a study of compression of steel tested at 1280 °C. They tested samples of various heights without lubrication and found that the shorter samples led to higher stress - strain curves as expected. A simulation of the compression test was performed using finite element analysis, in which sticking behaviour was assumed. For each height, the stress - strain curve calculated from the experimental load - deflection data was used as a first estimate of the true stress - strain curve. This led to a numerical load - deflection curve or force - strain curve $F_{\text{num}}(\varepsilon)$. Comparison between the experimental curve $F_{\text{exp}}(\varepsilon)$ and numerical curve $F_{\text{num}}(\varepsilon)$, led to the estimation of the correction factor as a function of strain, $c(\varepsilon)$, i.e.:

$$c_i(\varepsilon) = 1 - \frac{F_{\text{exp}}(\varepsilon)}{F_{i\,\text{num}}(\varepsilon)} \quad \text{for } i = 0, 1, 2 \ldots \tag{9}$$

where i is the number of iterations. The corrected stress - strain curve was then calculated from:

$$\sigma_{i+1}(\varepsilon) = \frac{\sigma_i(\varepsilon)}{1 + c_i(\varepsilon)} \tag{10}$$

The same iterative scheme was applied in this study for Gruyere and Mozzarella. The values of μ calculated from the analytical solution, i.e. 0.1 and 0.3 respectively, were assumed. The classical isotropic Coulomb friction model available in the software was used which defines the critical shear stress at which sliding of the surfaces starts as a fraction of the contact pressure, the fraction being equal to μ.

It was found that only one iteration was needed to bring $c(\varepsilon)$ to almost zero which is in agreement with Parteder and Bunten's results. The corrected stress - strain curves for all heights are shown in Fig. 13 for Mozzarella. The curve obtained from the lubricated experiments is also shown for comparison purposes. It is observed that the corrected curves corresponding to the various sample heights are in good agreement. In addition, the mean curve is very close to the lubricated curve. Very similar observations were made for Gruyere, hence the results are not shown.

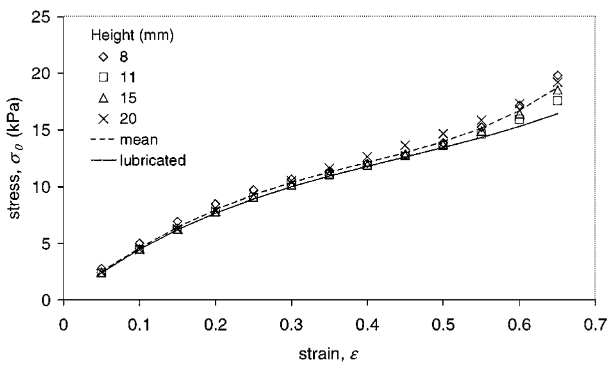


Figure 13 Comparison between the stress-strain curves predicted from the iterative finite element procedure and lubricated curve for Mozzarella.

TABLE I Material properties used in finite element models for Gruyere

Elastic modulus (kPa) 393.1	Poisson's ratio 0.49
Yield stress (kPa)	Plastic strain
19.7	0.000
31.2	0.021
39.4	0.050
46.2	0.082
52.4	0.117
58.0	0.153
63.1	0.190
66.5	0.231

TABLE II Material properties used in finite element models for Mozzarella

Elastic modulus (kPa) 48.0	Poisson's ratio 0.49
Yield stress (kPa)	Plastic strain
2.4	0.000
4.5	0.007
6.2	0.020
7.7	0.039
9.0	0.062
10.1	0.089
11.1	0.118
12.0	0.150
12.8	0.183
13.7	0.216
14.5	0.248
15.4	0.279
16.4	0.308

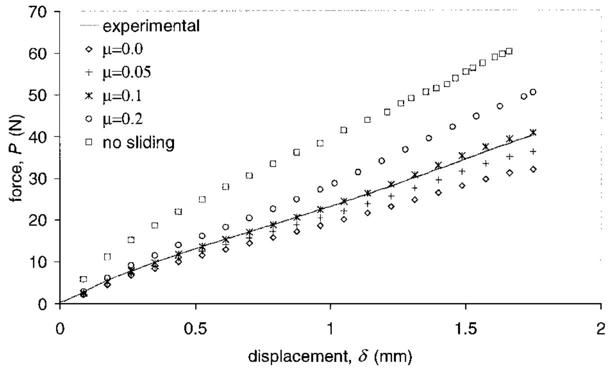


Figure 14 Comparison between numerical and unlubricated experimental load displacement curves for 5 mm Gruyere.

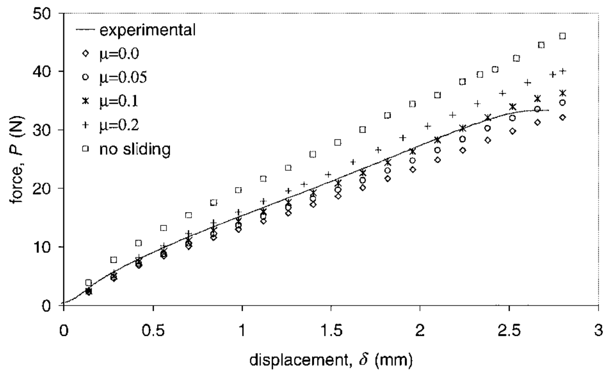


Figure 15 Comparison between numerical and unlubricated experimental load displacement curves for 8 mm Gruyere.

4.2. Determination of coefficient of friction

In the above section, pre-calculated values of μ were used in the simulation analysis to derive the stress - strain curve. In this section, the opposite procedure is followed, i.e. the stress - strain curve determined from the lubricated experiments (Figs 3 and 4) was used to define the mechanical behaviour of the cheese thus enabling the value of μ to be derived. This is the value of μ which leads to an agreement between the numerical and the unlubricated experimental load - deflection curves.

Tables I and II show the elastic modulus and yield stress as a function of plastic strain for Gruyere and Mozzarella.

The effect of various options in the software regarding the definition of the mechanical interaction between the contact surfaces was examined. Firstly, the case was tested where no sliding occurs once contact has been established. This implies that the cylinder and the rigid surface are perfectly adhered to each other. Secondly, the Coulomb friction model was used with μ set to values satisfying $0.0 \leq \mu \leq 0.4$. Note that $\mu = 0$ signifies frictionless conditions.

The numerically derived P - δ data for a Gruyere cylinder of height 5 mm are shown in Fig. 14. Results from five separate simulations are shown corresponding to $\mu = 0$, $\mu = 0.05$, $\mu = 0.1$, $\mu = 0.2$ and the case when the contact surfaces are sticking. The anticipated effect of increasing load with increasing coefficient of friction is observed. On the same plot, the average P - δ data measured from the experiments without lubrication are also shown. These are the data that were used with Equations 1 and 2 to obtain the plots shown in Fig. 1. It is obvious that the coefficient of friction which gives good agreement between the numerical and the experimental curves is approximately 0.1.

The comparison between the numerical predictions and the experimental data for the 8 mm samples is shown in Fig. 15. The estimated coefficient of friction is again approximately 0.1. The same value of μ was obtained from the results corresponding to the 11 mm samples (not shown). It was observed that the spread of the numerical curves corresponding to the various frictional conditions decreased considerably with increasing sample height. Therefore, the curves corresponding to the heights of 15 mm and 20 mm were too close to each other to enable estimation of μ and hence they are not shown. To summarise, it is concluded that the experimental data from non-lubricated experiments can be accurately reproduced by choosing a single value of μ, i.e. $\mu = 0.1$, for all sample heights and throughout the applied normal load range. In addition, this value of μ is in agreement with the value of μ estimated from the analytical solution ($\mu = 0.13$).

A similar procedure was followed for Mozzarella and the data for the heights of 8 and 11 mm are shown in Figs 16 and 17 respectively. From both of these plots, the coefficient of friction which gives an agreement between the numerical and experimental P - δ data is estimated to be between 0.2 and 0.3. This agrees with the analytical solution for μ. As in the case of Gruyere, the data corresponding to the 15 and 20 mm heights could not be used to derive an estimate of μ as there is very little effect of friction at these heights.

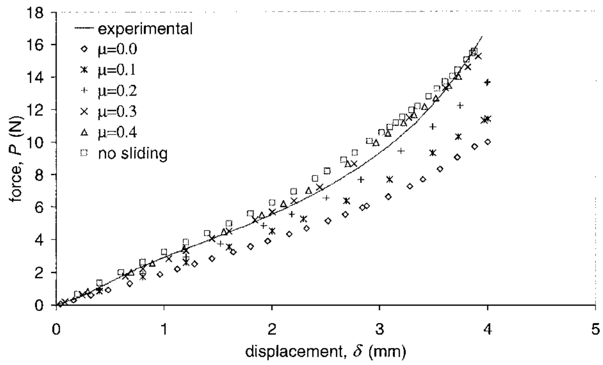


Figure 16 Comparison between numerical and unlubricated experimental load displacement curves for 8 mm Mozzarella.

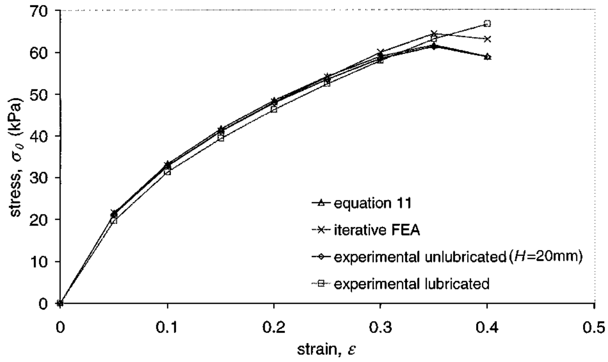


Figure 18 Comparison between lubricated test results, 20 mm unlubricated test results and predicted stress-strain curves for Gruyere.

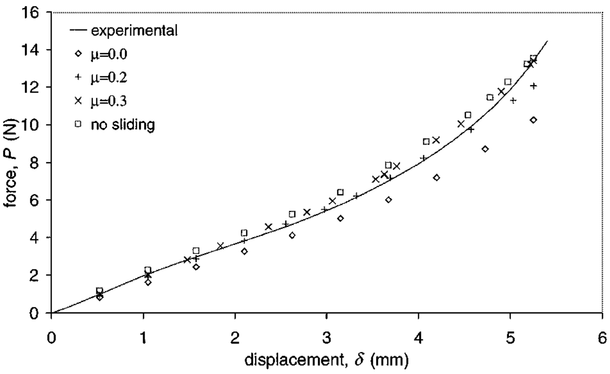


Figure 17 Comparison between numerical and unlubricated experimental load displacement curves for 11 mm Mozzarella.

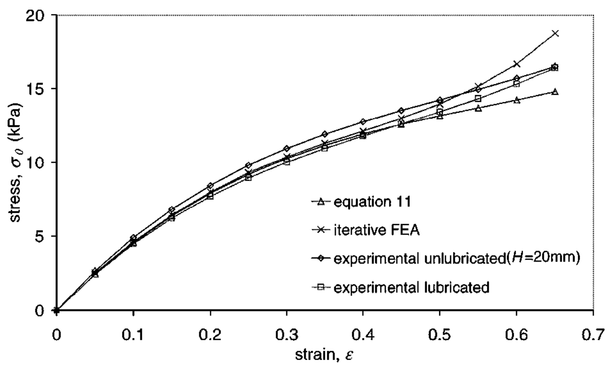


Figure 19 Comparison between lubricated test results, 20 mm unlubricated test results and predicted stress-strain curves for Mozzarella.

5. Discussion

The main problem in the analytical solution for the true stress - strain curve is that it relies on extrapolation which is an inherently inaccurate process. This is especially true for food materials because the experimental data are prone to scatter. The inaccuracy would decrease if data corresponding to taller samples were available. For example, if data from 40 mm tall samples were available, an extra point at $1/H = 0.025$ would be plotted in Fig. 8. However, practical problems render such tests not feasible, i.e. it is quite difficult to produce tall samples with an accurate cylindrical geometry. In addition, undesirable buckling effects arise at such large heights.

From Figs 1 and 2, it is evident that the difference between successive stress - strain curves corresponding to different heights reduces as the height increases. Therefore, as the sample height approaches infinity, the difference between successive stress - strain curves would be almost zero. Mathematically, this would correspond to a minimum point in the curve of p vs. $1/H$ at $1/H = 0$. Therefore, it was decided to fit a quadratic polynomial to the p vs. $1/H$ data of the form:

$$p = \sigma_0 + B\left(\frac{1}{H}\right)^2 \tag{11}$$

where B is a constant. Note that Equation 11 is incompatible with the analytical solution, i.e. Equation 8. As a result, no calculations of μ are possible. Nevertheless, the data were re-analysed and the extrapolated stress as a function of strain is shown in Figs 9 and 10. It is apparent that Equation 11 yields more accurate stress - strain curves than Equations 5 and 8 for both cheeses.

A comparison between the stress - strain curves predicted from the extrapolation procedure of Equation 11 and the iterative finite element procedure is shown in Figs 18 and 19 for Gruyere and Mozzarella respectively. Equation 11 is chosen as it was found to be the most accurate from all the proposed analytical expressions. The data from the lubricated experiments and the unlubricated experiments on 20 mm tall samples are also shown. The latter are plotted in order to examine whether there is a great benefit in using any scheme to extract the true stress - strain curves from unlubricated experimental data. In other words, the results from the tallest samples, i.e. $H = 20$ mm, might be almost as accurate and preferable as they require no further analysis. Indeed, it is observed that there is very little difference between the predictive scheme results and the 20 mm unlubricated data for both cheeses. Therefore it is concluded that for the two cheeses examined in this study, there is not a big gain in accuracy when the analytical or iterative numerical schemes are used.

Contrary to expectation, it is observed in Fig. 18 that the lubricated curve is higher than the unlubricated curve at strains larger than 0.35. This is due to the non-uniform stress distribution in the barrelled samples which leads to localised highly stressed zones. As a result, non-lubricated samples fail at smaller applied strains than lubricated samples. This was observed experimentally. The argument is further supported by the apparent shift of the maximum point in the curves to the left, i.e. to smaller strains. The maximum point marks the onset of cracking and the associated decrease in load. In addition, in Fig. 19, the lubricated curve for Mozzarella seems to approach the unlubricated curve as strain increases. This could be due to the loss of effectiveness of the lubricant at very large strains, i.e. the lubricant is essentially squeezed out of the contact area.

The agreement between the numerical prediction and the analytical solution regarding the coefficient of friction, μ, is encouraging. Determining friction in food materials is of paramount importance because many industrial processes such as cutting and shredding as well as textural attributes are greatly influenced by friction. There was however a variation in the analytical values of μ with strain, especially at small values of strain (Figs 11 and 12). This is because the mean stress versus $1/H$ curve at small strains is very flat (see Fig. 8). As μ is calculated from the slopes of this curve, a larger error is introduced. For Gruyere, a noticeable increase in μ was also observed for the larger strains. This was because the samples had already failed by this point and therefore the data at the large strains should not have been used in further calculations.

In order to examine the effect of the scatter inherent in experimental data on the analytically calculated values of μ, it was decided to repeat the calculation of μ using the data derived from the numerical simulations instead of experimental data. The calculations were performed using Equations 5, 7 and 8 as before. Numerical data corresponding to simulations with μ set to 0.1 and 0.3 were used for Gruyere and Mozzarella respectively. The results are shown in Figs 20 and 21. It is observed that Equation 5 leads to fairly constant μ values with averages of 0.10 and 0.32 for Gruyere and Mozzarella respectively. Therefore Equation 5 enables

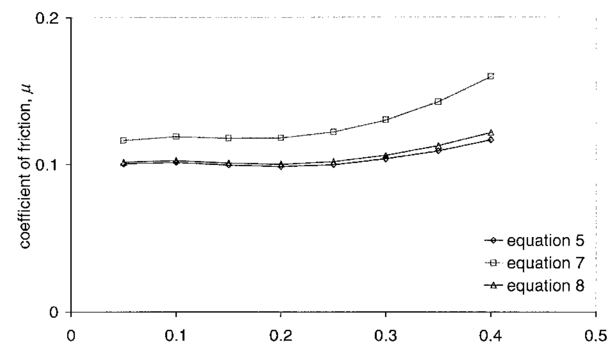


Figure 20 Prediction of μ from numerical data for Gruyere.

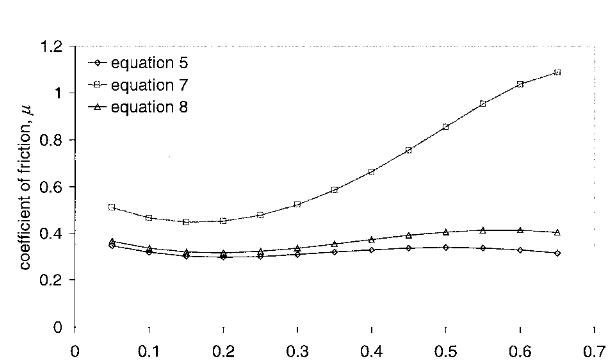


Figure 21 Prediction of μ from numerical data for Mozzarella.

back calculation of μ accurately. Equation 7 overestimates μ especially in the case of Mozzarella. This is expected as the linear approximation in Equation 7 will lead to a larger error for larger values of μ and ε. Equation 8 also leads to an overestimated μ for Mozzarella at large strains even though by a much smaller amount than Equation 7. The predictions of μ when experimental data are used (Figs 11 and 12) are less accurate due to the experimental scatter. Therefore, it is apparent that if this scheme is to be used to determine μ, a sufficiently large amount of accurate experimental data are needed.

The smaller value of μ for Gruyere than for Mozzarella could be attributed to the differing fat content of the cheeses; Mozzarella has a fat content of 23% where as the corresponding value for Gruyere is 32%. It seems that the larger fat content in Gruyere led to a reduction in friction and hence μ.

6. Conclusion

Superlube was found to eliminate friction in uniaxial compression tests of both Gruyere and Mozzarella. When no lubricant was used, the stress-strain curves appeared higher for decreasing sample heights. The samples barrelled as opposed to the uniform deformations observed when Superlube was used.

The accuracy of the analytical solution for the homogeneous compression of a flat circular disk in the presence of friction was examined. The analytical solution (Equation 5) as well as approximations (Equations 7 and 8) were used together with the stress - strain data obtained from the experiments without lubrication, in order to derive the true stress - strain curves. It was found that all equations led to stresses which were underestimated, with Equation 7 being the worst of the three. However, the stress - strain curves calculated using Equation 11 were very close to the lubricated curves. This equation is based on the assumption that there is a minimum in the mean stress versus $1/H$ curve at $1/H = 0$.

An iterative finite element analysis procedure was also used to derive the stress-strain curve derived from unlubricated test data. The values of μ calculated from the analytical solution were assumed, i.e. $\mu = 0.1$ for Gruyere and $\mu = 0.3$ for Mozzarella. It was found that the predicted stress - strain curves after the first iteration were very close to the curves measured from the lubricated experiments.

However, it was shown that for the two cheeses examined in this study, there is not a big gain in accuracy when the analytical or iterative numerical schemes are used. This is because the curve corresponding to the tallest sample from the unlubricated tests is close to the lubricated curve. Therefore, it is concluded that in cases where it is not possible to find a lubricant which will eliminate friction completely, the true stress-strain curves can be determined from unlubricated tests by testing samples of increasing heights until the difference between consecutive curves is negligible. The curve corresponding to the tallest sample can then be taken to represent the true stress-strain curve. If size or shape limitations do not allow testing of sufficiently tall samples, quadratic extrapolation of the results may then be performed (Equation 11). Alternatively, an iterative finite element analysis can be used. The latter is a more accurate but more time consuming method than the extrapolation procedure. In addition it requires that the coefficient of friction, μ, is known. The latter can be calculated using an analytical scheme (Equation 5). It was shown that the μ values calculated from Equation 5 agreed well with the values predicted from the finite element analysis. Interestingly, the same analytical scheme led to very accurate μ values when numerical instead of experimental data were used. This highlights the fact that great accuracy in experimental data is needed in order to determine the coefficient of friction with any level of confidence.

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